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The Cauchy stress tensor for a material subject to an isotropic internal constraint

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Abstract. The Cauchy stress tensor, T_{ij}, is considered for an elastic material which is subject to any internal isotropic constraint, apart from the constraint of incompressibility. For a given strain, it is seen that if in a given basis one of the eigenvectors of the stress tensor has a zero component, say the αth component, and if the arbitrary scalar term in the stress may be chosen so that $T_{\alpha\beta}$ ($\alpha \neq \beta$) shear stress component is zero, then the stress tensor has a double eigenvalue. This means that there is a great simplification in the stress field. The given strain may be maintained experimentally by a simple tension superimposed upon a hydrostatic stress field. Examples are presented for Bell-constrained and Ericksen-constrained materials.

Key words: stress, internal constraints, Bell and Ericksen materials, finite strains.

1. Introduction

The Cauchy stress tensor is real and symmetric, possessing, in general, three eigenvalues, the principal stresses, with corresponding orthogonal eigenvectors, the principal axes of stress. In an incompressible material all of the principal stresses contain precisely the same term, $-p$, which is the arbitrary hydrostatic pressure. In his work on the finite deformations of isotropic incompressible elastic materials Rivlin [1, pp. 23–142] recognized the arbitrariness in the choice of p and exploited it to make a particular plane free of normal traction, thereby simplifying the situation. Because precisely the same term occurs in all the principal stresses, the choice of p cannot affect the general structure of the stress tensor. Now consider materials subject to other isotropic internal constraints such as Bell materials or Ericksen materials. For a Bell material, for example, the Cauchy stress contains the term $-p\mathbf{V}$, where $\mathbf{V}^2 = \mathbf{B}$ is the left Cauchy–Green strain tensor. The principal stresses in turn contain $-p\lambda_1$, $-p\lambda_2$, $-p\lambda_3$, where λ_1, λ_2, λ_3 are the principal stretches. If it is assumed that the stretches are different, it is clearly possible to choose p so that two of the principal stresses are equal and thus the character of the stress tensor is changed to that of a simple tension superimposed upon a hydrostatic stress. This is the central idea in this note. The purpose is to show how in certain circumstances the structure of the stress tensor, in the basis in which the deformation and the strain are described, may be very simply altered to that of simple tension superimposed upon a hydrostatic stress, which is a great simplification for the experimentalist in maintaining the deformation. No particular symmetry of the material is assumed.

2. Basic equations

We assume that the components of the Cauchy stress $\mathbf{T}$ are T_{ij} with respect to a rectangular Cartesian system with orthonormal basis $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. Thus, for example, $T_{12} = \mathbf{i} \cdot \mathbf{Tj}$.

Let $\mathbf{F}$ be the deformation gradient from the reference configuration and let its polar decomposition be given by $\mathbf{F} = \mathbf{RU} = \mathbf{VR}$ where $\mathbf{R}^T\mathbf{R} = \mathbf{I}$. Then, the left and right Cauchy–Green strain tensors are given, respectively, by $\mathbf{B} = \mathbf{FF}^T = \mathbf{V}^2$, and $\mathbf{C} = \mathbf{F}^T\mathbf{F} = \mathbf{U}^2$. The principal invariants of $\mathbf{B}$ are denoted by I_1, I_2, I_3 where $I_1 = \operatorname{tr}\mathbf{B}$, $2I_2 = (\operatorname{tr}\mathbf{B})^2 - \operatorname{tr}\mathbf{B}^2$, $I_3 = \det\mathbf{B}$, and the principal invariants of $\mathbf{V}$ are similarly denoted $I_1^V = \operatorname{tr}\mathbf{V}$, $2I_2^V = (\operatorname{tr}\mathbf{V})^2 - \operatorname{tr}\mathbf{V}^2$, $I_3^V = \det\mathbf{V}$.

For illustrative purposes we consider the particular cases of Bell and Ericksen materials. Similar considerations apply to any material that is subject to an isotropic internal constraint, apart from an incompressible material.

For a Bell-constrained isotropic material [2, 3], the internal constraint is given by $\operatorname{tr}\mathbf{V} = 3$, and the constitutive equation by

$$T = -p\mathbf{V} + \omega_0\mathbf{I} + \omega_2\mathbf{B}, \tag{1}$$

or, in components,

$$T_{ij} = -pV_{ij} + \omega_0\delta_{ij} + \omega_2 B_{ij}, \tag{2}$$

where p is to be determined from the equilibrium equations and boundary conditions and ω_0, ω_2 are functions of I_2^V and I_3^V. For an isotropic Ericksen material [4] the internal constraint is given by $\operatorname{tr}\mathbf{B} = 3$ and, the constitutive equation is

$$\mathbf{T} = -p\mathbf{B} + \beta_0\mathbf{I} + \beta_{-1}\mathbf{B}^{-1}, \tag{3}$$

where β_0, β_{-1} are functions of I_2 and I_3.

We note that, because both $\mathbf{V}$ and $\mathbf{B}$ are positive definite, all normal components of $\mathbf{T}$ will include p. Also, if, for example, for Bell materials $V_{12} \neq 0$, then the shear stress component T_{12} will also include the reaction scalar. Then, in this case, p may be chosen so that either one normal component of traction or the shear stress component T_{12} is zero. Indeed, in this regard, there is a greater variety of possibilities with a Bell or an Ericksen material than with an incompressible material.

3. Basic result

Consider any material subject to an isotropic internal constraint, other than incompressibility. Suppose that $\mathbf{T}$ has three eigenvalues t_1, t_2, t_3 and three unit eigenvectors with Cartesian components e_i, m_i, s_i with respect to the basis $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, so that

$$T_{ij}e_j = t_1 e_i, \qquad T_{ij}m_j = t_2 m_i, \qquad T_{ij}s_j = t_3 s_i. \tag{4}$$

The eigenvectors $\mathbf{e}$, $\mathbf{m}$, $\mathbf{s}$ form an orthonormal triad. Also, note that at this stage p has not been chosen. Now suppose that one of the eigenvectors $\mathbf{e}$, $\mathbf{m}$ or $\mathbf{s}$ has a zero component with respect to the basis $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. Thus, for example, assume that $e_1 = 0$, or put in another way,

$$\mathbf{i} \cdot \mathbf{Te} = 0. \tag{5}$$

Suppose now that p may be chosen so that the shear stress component T_{12} is zero, thus

$$\mathbf{i} \cdot \mathbf{Tj} = 0. \tag{6}$$

It now follows from (5) and (6) that

$$\mathbf{Ti} = \mu \mathbf{i}, \tag{7}$$

for some μ, so that **i** is an eigenvector of **T** with eigenvalue μ. Also, of course, from Equation (7)

$$\mathbf{i} \cdot \mathbf{Tk} = 0. \tag{8}$$

Now consider $(4)_2$ and $(4)_3$:

$$m_1\mathbf{Ti} + m_2\mathbf{Tj} + m_3\mathbf{Tk} = t_2(m_1\mathbf{i} + m_2\mathbf{j} + m_3\mathbf{k}),$$

$$s_1\mathbf{Ti} + s_2\mathbf{Tj} + s_3\mathbf{Tk} = t_3(s_1\mathbf{i} + s_2\mathbf{j} + s_3\mathbf{k}).$$

Using (6) and (8) we observe that $\mu = t_2$ and $\mu = t_3$. Thus, the structure of **T** is given by

$$\mathbf{T} = t_2\mathbf{I} + (t_1 - t_2)\mathbf{e} \otimes \mathbf{e} = t_1\mathbf{e} \otimes \mathbf{e} + t_2(\mathbf{T} - \mathbf{e} \otimes \mathbf{e}).$$

As a result, for a circular cylindrical specimen of material with axis along **e**, the stress consists of the simple tension t_1 along **e** and an all round tension t_2 on the lateral surfaces of the cylinder. Alternatively, the stress consists of a hydrostatic tension t_2 superposed on the simple tension $(t_1 - t_2)$ along **e**.

3.1. REMARK 1. The result presented here is a special case of the following result in linear algebra. Given a real symmetric 3×3 matrix **T** and two vectors **i**, **j**, not eigenvectors of **T** and such that $\mathbf{i} \cdot \mathbf{j} = 0$. Now suppose that **i** is orthogonal to an eigenvector of **T** and also that $\mathbf{i} \cdot \mathbf{Tj} = 0$. Then at least two eigenvalues of **T** coincide.

3.2. REMARK 2. It is well known, [5], for isotropic materials, subject to a homogeneous deformation, that is always possible to find three universal relations which are the expression of the coaxiality of the stress and the strain. In the case of a constrained material choosing the unknown scalar related to the reaction stress via the equilibrium equations and the boundary conditions leads to a fourth universal relation, which is a link between the eigenvalues of the stress. In our case the fourth universal relation is exactly the expression of the coincidence of two eigenvalues for the stress [6].

3.3. REMARK 3. Except for incompressible materials, the result of this note extends to all materials subject to an isotropic constraint for which it is possible to make an appropriate choice of the reaction scalar. In the general case the constraint equation is

$$\gamma(I_1 - 3, I_2 - 3, I_3 - 1) = 0, \tag{9}$$

where γ is a smooth function of the principal invariants of **C**, with a simple zero root[1]. Now for the stress tensor we have

$$\mathbf{T} = -\pi \mathbf{F} \partial_{\mathbf{C}} \gamma \mathbf{F}^{\mathbf{T}} + \mathbf{T}_E, \tag{10}$$

[1] Also for the Bell material it is possible to get a representation in terms of these arguments, because $I_1^V = 3$, $2I_2^V = 9 - I_1$, and $I_1^2 - 18\, I_1 - 4I_2 - 24\sqrt{I_3} + 81 = 0$, in which the first is Bell's constraint and the last follows by application of the Cayley–Hamilton theorem.

where π is the reaction scalar and $\mathbf{T}_E$ the extra stress. For isotropic materials a representation formula is [7]

$$\mathbf{T} = -\pi\mathbf{A} + \beta_0\mathbf{I} + \beta_1\mathbf{B} + \beta_{-1}\mathbf{B}^{-1},$$

where

$$\mathbf{A} = \left(I_2\frac{\partial\gamma}{\partial I_2} + I_3\frac{\partial\gamma}{\partial I_3}\right)\mathbf{I} + \frac{\partial\gamma}{\partial I_1}\mathbf{B} + I_3\frac{\partial\gamma}{\partial I_2}\mathbf{B}^{-1}, \tag{11}$$

and β_i are the response coefficients. These are functions of the invariants, subject to the constraints. Obviously, $\mathbf{A}$ and $\mathbf{B}$ are coaxial ($\mathbf{AB} = \mathbf{BA}$) and so are $\mathbf{A}$ and $\mathbf{V}$, but the eigenvalues of these tensors are not simply related. The details, with a discussion on special cases, may be found in [7]. In particular, from (11) it is simple to check that the only materials with a spherical reaction stress tensor are the incompressible ones and for these materials we do not have the possibility of choosing the reaction scalar to render zero the off diagonal terms of the the stress tensor. This is why the qualification ‘apart from incompressibility’ is used in the statement of the main result. Indeed, it is clear for an incompressible material that the reaction scalar pressure may not be chosen so that two of the principal stress are equal.

4. Examples

No particular symmetry of the material has been assumed in deriving the basic result. However, for the sake of simplicity the examples we now present are all for isotropic materials.

4.1. Homogeneous deformation of a Bell material

For illustrative purposes let us consider an isotropic Bell material and suppose that at a point in the material $\mathbf{V}$ is given by

$$\mathbf{V} = \frac{3}{19}\begin{pmatrix} 7 & -1 & 1 \\ -1 & 6 & -2 \\ 1 & -2 & 6 \end{pmatrix}, \tag{12}$$

a matrix with eigenvectors

$$\mathbf{e} = \frac{1}{\sqrt{2}}(0, \quad 1, \quad 1), \quad \mathbf{m} = \frac{1}{\sqrt{3}}(1, \quad -1, \quad 1), \quad \mathbf{s} = \frac{1}{\sqrt{6}}(2, \quad 1, \quad -1),$$

and eigenvalues 12/19, 27/19, 18/19. Note that one of the eigenvectors has a zero component. From the constitutive Equation (1) we have

$$\begin{aligned}\mathbf{T} \;=\; & -p\frac{3}{19}\begin{pmatrix} 7 & -1 & 1 \\ -1 & 6 & -2 \\ 1 & -2 & 6 \end{pmatrix} \\ & +\omega_0\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \omega_2(\frac{3}{19})^2\begin{pmatrix} 51 & -15 & 15 \\ -15 & 41 & -25 \\ 15 & -25 & 41 \end{pmatrix}.\end{aligned}$$

Now to make $T_{12} = 0$ choose:

$$p = \frac{45}{19}\omega_2,$$

and then

$$\mathbf{T} = \omega_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{\omega_2}{361} \begin{pmatrix} -486 & 0 & 0 \\ 0 & -441 & 45 \\ 0 & 45 & -441 \end{pmatrix},$$

and so the principal stresses are $t_1 = t_3 = \omega_0 - \frac{486}{361}\omega_2$, and $t_2 = \omega_0 - \frac{396}{361}\omega_2$, with

$$\mathbf{T} = \left(\omega_0 - \frac{396}{361}\omega_2\right)\mathbf{I} - \frac{90}{361}\omega_2 \mathbf{e} \otimes \mathbf{e},$$

where $\mathbf{e} = \frac{1}{\sqrt{2}}(0, 1, 1)$.

We note that the eigenvectors of **T** are **e** and any vector orthogonal to **e**. The eigenvectors of **V** are eigenvectors of **T**, but not all the eigenvectors of **T** are eigenvectors of **V**.

4.2. Generalized plane deformation

Consider the deformation described in a rectangular Cartesian coordinate system:

$$x = x(X, Y), \quad y = y(X, Y), \quad z = \lambda Z,$$

where the particle initially at (X, Y, Z) is displaced to (x, y, z) and λ is a constant. The strain **V** or **B** associated with this deformation has an eigenvector $(0, 0, 1)$. The pressure term p for an Ericksen material or a Bell-material may be chosen so that $T_{12} = 0$ and consequently the stress consists of a simple tension in the z-direction and the all round tension $T_{11} = T_{22}$ in the $x - y$ plane.

For isotropic materials it is possible to see this directly. Consider the universal relations $\mathbf{BT} = \mathbf{TB}$ (or $\mathbf{VT} = \mathbf{TV}$) that result from the coaxiality of the stress with the strain for these materials [5]. In the present case, with (3) in mind, two of these universal relations are trivial: $T_{13} = 0$ and $T_{23} = 0$, the third being given by[2]

$$B_{12}(T_{11} - T_{22}) = (B_{11} - B_{22})T_{12}, \qquad \text{or} \qquad V_{12}(T_{11} - T_{22}) = (V_{11} - V_{22})T_{12}.$$

Thus, it is clear that if $T_{12} = 0$, then $T_{11} = T_{22}$.

An explicit example of generalized plane deformation is the Singh–Pipkin deformation [8], described in terms of cylindical polar coordinates (r, θ, z) by

$$r = AR, \theta = B\ \log(R/R_0) + C\Theta, z = DZ,$$

[2] The possibility of B_{12} or V_{12} being zero is ruled out because it is assumed that p may be chosen so that $T_{12} = 0$, the coefficient of p in the expression of this stress component being either B_{12} or V_{12}.

which describes the inflation, bending, extension and azimuthal shearing of an annular wedge. Here the coefficients A, B, C, D, R_0 are constants. Now in physical components, we have [9]

$$\mathbf{V} = \begin{pmatrix} A(C+1)K & ABK & 0 \\ ABK & AK[C(C+1)+B^2] & 0 \\ 0 & 0 & D \end{pmatrix},$$

where $K = \left(\sqrt{(C+1)^2 + B^2}\right)^{-1}$. It is easy to check that the strain invariants are constant. This deformation has been shown to be admissible in all isotropic materials subject to an internal isotropic constraint[3] see [6] and [7]. All the corresponding details for Bell materials have been worked out in [9] where it is shown that the nonzero physical stress components are

$$T_{rr} = pAK(C+1) + \omega_0 + A^2\omega_2,$$

$$T_{\theta\theta} = pAK[B^2 + C(C+1)] + \omega_0 + A^2(B^2 + C^2)\omega_2,$$

$$T_{zz} = pD + \omega_0 + D^2\omega_2, \qquad T_{r\theta} = ABK\left(p + \frac{A}{K}\omega_2\right).$$

So, if $p = -\frac{A}{K}\omega_2$, then $T_{r\theta} = 0$, and automatically from our basic result we have $T_{rr} = T_{\theta\theta}$, as is readily checked.

4.3. Equibiaxial stretch and sinusoidal shear

Consider the deformation [10]

$$x = AX + \sin\widetilde{Y}, \quad y = D\widetilde{Y}, \quad z = AZ - \cos\widetilde{Y},$$

where $\widetilde{Y} = kY$ and k, A, D are constants. This deformation has also been shown to be admissible in Bell materials by Beatty and Hayes [9] and it has been shown to be admissible in all isotropic materials constrained with an isotropic constraint (with the exception of incompressibility) in [6]. In the case of Ericksen materials we have

$$\mathbf{B} = \begin{pmatrix} A^2 + \cos^2\widetilde{Y} & D\cos\widetilde{Y} & \sin\widetilde{Y}\cos\widetilde{Y} \\ D\cos\widetilde{Y} & D^2 & D\sin\widetilde{Y} \\ \sin\widetilde{Y}\cos\widetilde{Y} & D\sin\widetilde{Y} & A^2 + \sin^2\widetilde{Y} \end{pmatrix},$$

$$\mathbf{B}^{-1} = \begin{pmatrix} \dfrac{1}{A^2} & -\dfrac{1}{D}\dfrac{\cos\widetilde{Y}}{A^2} & 0 \\ -\dfrac{1}{D}\dfrac{\cos\widetilde{Y}}{A^2} & \dfrac{1}{D^2} + \dfrac{1}{A^2D^2} & -\dfrac{1}{D}\dfrac{\sin\widetilde{Y}}{A^2} \\ 0 & -\dfrac{1}{D}\dfrac{\sin\widetilde{Y}}{A^2} & \dfrac{1}{A^2} \end{pmatrix}.$$

[3] Obviously the constraint introduces a link between the various coefficients which is different from constraint to constraint.

This deformation is admissible if and only if we have

$$p = -\frac{\beta_{-1}}{D^2 A^2}.$$

Here a choice of p is no longer available. Even so, this value of p ensures $T_{12} = 0$, and our result is still valid because one eigenvector of **B** is $(\sin \widetilde{Y}, 0, -\cos \widetilde{Y})$.

The nonzero physical stress components for an Ericksen material are then given by

$$T_{11} = \frac{\beta_{-1}}{D^2 A^2}\left(A^2 + \cos^2 \widetilde{Y}\right) + \beta_0 + \frac{\beta_{-1}}{A^2}, \qquad T_{22} = \frac{\beta_{-1}}{A^2} + \beta_0 + \beta_{-1}\left(\frac{1}{D^2} + \frac{1}{A^2 D^2}\right),$$

$$T_{33} = \frac{\beta_{-1}}{D^2 A^2}\left(A^2 + \sin^2 \widetilde{Y}\right) + \beta_0 + \frac{\beta_{-2}}{A^2}, \qquad T_{13} = \frac{\beta_{-1}}{D^2 A^2} \sin \widetilde{Y} \cos \widetilde{Y}.$$

The principal stresses are

$$t_1 = t_2 = -\beta_{-1} D^2 A^2 + \beta_0, \qquad t_3 = -\beta_{-1}(A^2 + D^2 A^2) + \beta_0,$$

so that the stress again consists of the simple tension t_3 in the z direction and the all round tension $t_1 = t_2$ in the xy−plane.

Discussion of results

The most familiar internal constraint in the finite elasticity theory is that of incompressibility, used in the description of the mechanical behaviour of rubber. Another familiar constraint, though not isotropic, is that of inextensibility in a certain direction, used in the description of the mechanical behaviour of fibre-reinforced materials such as composites. New isotropic internal constraints such as those of Bell and Ericksen have also been used.

In the case of incompressible materials the mathematical effect of the constraint is that the constitutive equation relating the stress with the strain contains an arbitrary hydrostatic-pressure term in the stress tensor, so that the Cauchy stress is determined by the strain only to within a hydrostatic pressure. However, as pointed out by Rivlin (see [1]), this arbitrariness in the stress may be exploited in the solution of problems, particularly when the normal component of traction across one surface may be chosen to constant, in particular zero. For the experimentalist this means there is a simplification in the system of forces required to maintain the specimen in the particular state of deformation.

In this paper isotropic internal constraints, other than incompressibility, have been considered for isotropic elastic materials. In this case the mathematical effect is that the constitutive equation relating the stress with the strain contains a certain strain tensor multiplied by an arbitrary scalar, so that the Cauchy stress is determined by the strain only to within a term consisting of the certain strain tensor multiplied by the arbitrary scalar. It was shown that this arbitrariness in the Cauchy stress may be exploited to simplify problems, with a far greater range of possibilities than are available for incompressible materials. The key point is that, whereas for incompressible materials the arbitrary term in the Cauchy stress is a hydrostatic pressure, *viz.* a scalar multiple of the unit tensor, which means that only a normal component of traction may be chosen arbitrarily, in other than incompressible materials subject to an isotropic constraint the arbitrary term in the Cauchy stress is a second-order tensor which is not a scalar multiple of the unit tensor, so that it may be possible to choose arbitrarily either one component of normal traction or one shear component of traction, or a suitable combination. For the experimentalist this means that there is some simplification in the system of forces required to maintain a specimen in a particular state of deformation. (Such general considerations will, of course, apply in areas other than the finite strain of isotropic elastic materials.)

In particular, it has been shown, for materials subject to an isotropic constraint other than incompressibility that, if in a given basis, one of the eigenvectors of the stress tensor has a zero component and if the arbitrariness in the stress may be exploited to make a certain shear component of stress zero, then the stress tensor has a double eigenvalue. For the experimentalist this means that the given strain field in the material may be maintained by a hydrostatic pressure superposed on a simple tension.

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